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CONSTRUCTION OF SYMMETRIC THIRD ORDER DIFFERENCE OPERATORS ON A HILBERT SPACE

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Abstract. We have constructed a symmetric third order difference equation and studied the absolutely continuous spectrum of the self adjoint subspace extension generated using the subspace and decomposition theories. In particular, we have shown that under some decay and growth conditions, the self-adjoint subspace extension exists with discrete spectrum on $l^2(\mathbb{N})$ while the absolutely continuous spectrum contained on half line of multiplicity one exists on $l^2(\mathbb{Z})$ if we use decomposition technique. We have given a number of examples to reinforce our results.

Keywords. Odd order difference equations, symmetric difference equations, absolutely continous spectrum
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1. Introduction

In several papers, one of the authors and others, have studied the spectral theory of even-order difference operators [6, 5, 1, 3, 7, 18, 19, 20]. Recently, attention has shifted from studying even order differential equations to the odd higher order differential equations and difference operators. Hinton in [13] computed using the limit point criteria for differential equations the deficiency indices for odd order differential equations. Infact, Behncke and Hinton [4] analyzed the asymptotic behavior of eigenfunctions for a rather general class of odd-order differential operators on the half-line [4]. Consequently, it is natural to consider extending this kind of analysis to general odd-order difference operators. Difference equations are usually regarded as the discretization of the corresponding differential equations. It is well established from [7] that the results for symmetric differential operators generally correspond to those of their discrete counterparts when subject to similar growth and decay conditions. Based on current findings, most properties in the spectral theory of difference equations align with those of the corresponding differential equations, though a few significant ones

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